TEST CODE **02134020**

FORM TP 2017297

MAY/JUNE 2017

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 02

ALGEBRA, GEOMETRY AND CALCULUS

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of TWO questions.
3. Answer ALL questions from the THREE sections.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A

Module 1

Answer BOTH questions.

1. (a) Let p and q be two propositions

p : It is raining.

q : John is sick.

Write EACH of the statements below in terms of p and q.

(i) It is not raining or John is sick.

....

[1 mark]

(ii) If it is raining then John is not sick.

....

[1 mark]

(b) An operation * is defined on the set {1, 2, 3, 4} as shown in the following table.

*	1	2	3	4
1	2	4	1	3
2	4	3	2	1
3	1	2	3	4
4	3	1	4	2

(i) Prove that * is commutative.

[1 mark]

(ii) Show that the identity element of * is 3.

[2 marks]

GO ON TO THE NEXT PAGE

(c) The polynomial $f(x) = ax^3 + 9x^2 - 11x + b$ has a factor of $(x - 2)$ and a remainder of 12 when divided by $(x + 2)$.

(i) Show that $a = 2$ and $b = -30$.

[4 marks]

(ii) Hence, solve $ax^3 + 9x^2 - 11x + b = 0$.

[9 marks]

GO ON TO THE NEXT PAGE

(d) Use mathematical induction to prove that

$8 + 16 + 24 + 32 + \ldots + 8n = 4n(n + 1)$ for all $n \in \mathrm{N}$.

[7 marks]

Total 25 marks

2. (a) (i) Given that $a^2 + b^2 = 14ab$, prove that $\ln\left(\dfrac{a+b}{4}\right) = \dfrac{1}{2}(\ln a + \ln b)$.

[5 marks]

GO ON TO THE NEXT PAGE

(ii) Solve the equation $2^{-x} + 3(2^x) = 4$.

[Your response may be expressed in terms of logarithms.]

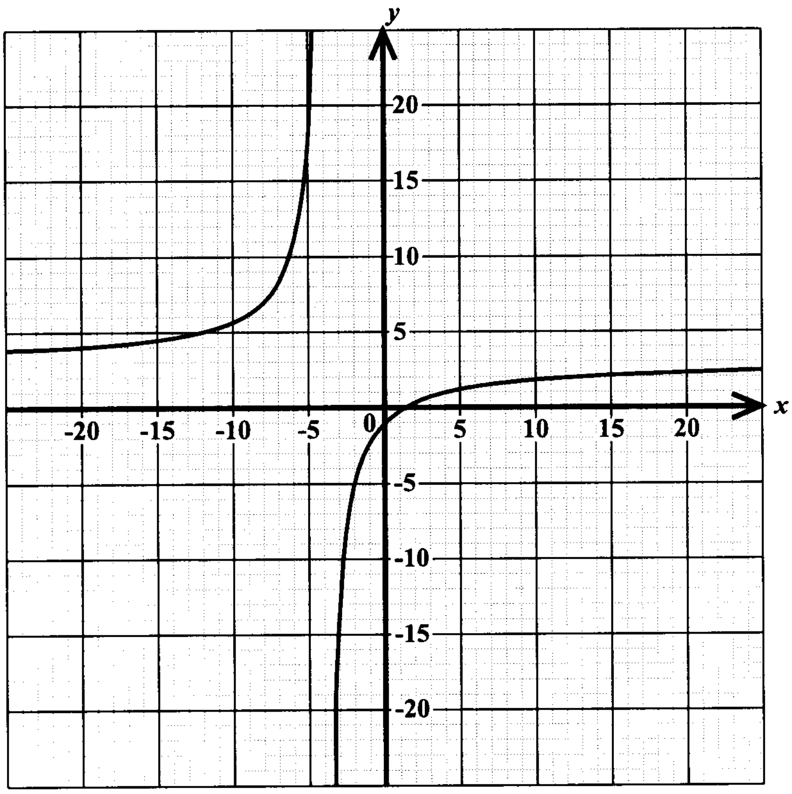
[6 marks]

(b) The following diagram shows the graph of the function $f(x) = \dfrac{3x-4}{x+4}$.

On the diagram,

(i) insert the asymptotes for the function f **[2 marks]**

(ii) sketch the graph of f^{-1}, the inverse of f showing the asymptotes for f^{-1}. **[4 marks]**



(c) Given that α, β and γ are the roots of the equation $x^3 + 3x + 2 = 0$, form an equation whose roots are βγ, αγ and αβ.

[8 marks]

Total 25 marks

SECTION B

Module 2

Answer BOTH questions.

3. (a) (i) Prove the identity

$$\tan(A + B) \equiv \frac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)}$$

[4 marks]

GO ON TO THE NEXT PAGE

(ii) Given that $\sin A = \frac{3}{5}$ and $\cos B = \frac{-1}{2}$ where angle A is acute and angle B is obtuse, express $\tan(A + B)$ in the form $a + b\sqrt{3}$, where a and b are real numbers.

[6 marks]

(b) Solve the equation $\sin^2\theta - 2\cos^2\theta + 3\cos\theta + 5 = 0$ for $0 \leq \theta \leq 4\pi$.

[6 marks]

(c) (i) Express $f(\theta) = 6 \cos \theta + 8 \sin \theta$ in the form $r \sin (\theta + \alpha)$ where $0 \leq \alpha \leq 90°$.

[3 marks]

(ii) Hence, or otherwise, find the general solution of $f(\theta) = 2$.

[6 marks]

Total 25 marks

4. (a) (i) The circle, C_1, with equation $x^2 + y^2 - 4x + 2y - 2 = 0$ and the circle C_2 have a common centre. Given that C_2 passes through the point $(-1, -2)$, express the equation of C_2 in the form $(x - h)^2 + (y - k)^2 = k$.

[3 marks]

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(ii) The equation of the line L_1 is $x + 3y = 3$. Determine whether L_1 is a tangent to the circle, C_1, in a (i) on page 16.

[7 marks]

(b) Let **P**(3, 1, 2) and **Q**(1, –2, 4).

(i) Express the vector $\overrightarrow{\mathbf{PQ}}$ in the form $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

[2 marks]

(ii) Determine the Cartesian equation of the plane which passes through the point **Q** and is perpendicular to $\overrightarrow{\mathbf{PQ}}$.

[6 marks]

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(c) The vector equations of two lines, L_1 and L_2, are:

$L_1 = -\mathbf{i} + \mathbf{j} - 2\mathbf{k} + \alpha\,(-2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$

$L_2 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k} + \beta\,(\mathbf{i} - \mathbf{j} + \mathbf{k})$

(i) Show that L_1 and L_2 intersect.

[5 marks]

(ii) Hence, determine the coordinates of the point of intersection of the two lines.

[2 marks]

Total 25 marks

SECTION C

Module 3

Answer BOTH questions.

5. (a) Determine the value of k for which

$$f(x) = \begin{cases} \dfrac{x^5 - 1}{x - 1}, & x \neq 1 \\ k, & x = 1 \end{cases}$$

is continuous for all values of x.

[4 marks]

(b) A curve, C, is described parametrically by the equations $x = 5t + 3$ and $y = t^3 - t^2 + 2$.

(i) Find $\frac{dy}{dx}$ in terms of t.

[3 marks]

(ii) Hence, determine all points of C such that $\frac{dy}{dx} = 0$.

[6 marks]

(c) (i) Given that $y = \sqrt{2 + 2x^2}$, show that

a) $y\dfrac{dy}{dx} - 2x = 0$

b) $\dfrac{d^2y}{dx^2} - \dfrac{4}{y^3} = 0.$

[9 marks]

GO ON TO THE NEXT PAGE

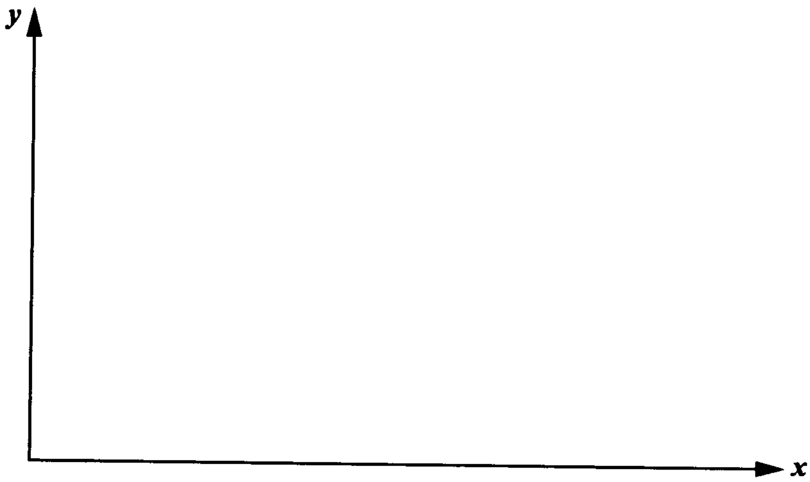
(ii) Hence, find the value of $\frac{d^2y}{dx^2}$ when $x = 0$.

[3 marks]

Total 25 marks

6. (a) Triangle PQR has vertices $P(0, 1)$ $Q(3, 3)$ and $R(4, 2)$.

(i) On the axes below, sketch triangle PQR.



[1 mark]

(ii) Determine the equations of EACH of the following:

- PQ
- QR
- PR

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[7 marks]

(iii) Hence, use integration to determine the area of triangle PQR.

[7 marks]

(b) The voltage in a circuit, V, satisfes the equation $\frac{\mathrm{d}V}{\mathrm{dt}} + \frac{V}{2.5} = 0$. Given that $V = 25$ volts when $t = 0$ seconds, write an expression for V in terms of t.

[5 marks]

(c) Given that $\int_{-1}^{3} [3f(x) + g(x)]\, dx = 5$ and $\int_{-1}^{3} [5f(x) - 2g(x)]\, dx = 1$, determine

- $\int_{-1}^{3} f(x)\, dx$
- $\int_{-1}^{3} g(x)\, dx$.

[5 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.