CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

20 MAY 2019 (a.m.)

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FILL IN ALL THE INFORMATION REQUESTED CLEARLY IN CAPITAL LETTERS.

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SUBJECT PURE MATHEMATICS – UNIT 1 – Paper 02

PROFICIENCY ADVANCED

REGISTRATION NUMBER | | | | | | | | | | |

SCHOOL/CENTRE NUMBER

NAME OF SCHOOL/CENTRE

CANDIDATE'S FULL NAME (FIRST, MIDDLE, LAST)

DATE OF BIRTH | D | D | M | M | Y | Y | Y | Y |

SIGNATURE _____

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FORM TP 2019305

TEST CODE 02134020

MAY/JUNE 2019

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 02

ALGEBRA, GEOMETRY AND CALCULUS

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of TWO questions.
3. Answer ALL questions from the THREE sections.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

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SECTION A

Module 1

Answer BOTH questions.

1. (a) The quadratic expression $f(x) = ax^2 + 12x + b$ is divisible by $x - 3$ and has a remainder of -27 when divided by $x + 6$.

(i) Show that the values of the constants a and b are 3 and -63, respectively.

[7 marks]

(ii) Hence, determine the factors of f.

[3 marks]

(b) Solve, for real values of x, the inequality $|3x - 4| \leq 6$.

[4 marks]

GO ON TO THE NEXT PAGE

(c) A binary operation is defined on the set of rational numbers by $a * b = \frac{ab}{2}$. Prove that $*$ is commutative.

[3 marks]

(d) Use mathematical induction to prove that $5^n - 1$ is divisible by 4 for $n \in \mathbf{N}$.

[8 marks]

Total 25 marks

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2. (a) Let $A = \{x : x \in \mathbf{R}, x \geq 1\}$.

A function $f : A \rightarrow \mathbf{R}$ is defined as $f(x) = x^2 - x$. Show that f is one-to-one.

[7 marks]

(b) Let $f(x) = 3x + 1$ and $g(x) = e^{3x}$.

(i) Determine f^{-1}.

[2 marks]

(ii) Determine $f^{-1} \circ g$.

[2 marks]

(c) Solve the equation $3 - \frac{4}{9^x} - \frac{4}{81^x} = 0$.

[7 marks]

(d) Let the roots of the equation $2x^3 - 5x^2 + 4x + 6 = 0$ be α, β and γ.

(i) State the values of $\alpha + \beta + \gamma$, $\alpha\beta + \alpha\gamma + \beta\gamma$ and $\alpha\beta\gamma$.

[3 marks]

(ii) Hence, or otherwise, determine the equation with roots α^2, β^2 and γ^2.

Note: $(\alpha\beta)^2 + (\alpha\gamma)^2 + (\beta\gamma)^2 = (\alpha\beta + \alpha\gamma + \beta\gamma)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)$

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

[4 marks]

Total 25 marks

SECTION B

Module 2

Answer BOTH questions.

3. (a) Solve the equation $5 \sec \theta - 2 \sec^2 \theta = \tan^2 \theta - 1$ for $-\pi \leq \theta \leq \pi$.

[7 marks]

(b) Let $f(x) = \cos x + 2 \sin x$.

(i) Express f in the form $R \sin(x + \alpha)$, where α is an acute angle.

[5 marks]

(ii) Hence, or otherwise, find the general solution of $\cos x + 2 \sin x = 0$.

[3 marks]

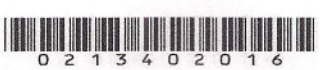
(iii) Determine the **minimum** value of $\frac{2}{2-f}$.

[3 marks]

(c) Prove the identity $\tan(A + B) - \tan A = \dfrac{\sin B}{\cos A \cos (A + B)}$.

[7 marks]

Total 25 marks



4. (a) A circle with equation $x^2 + y^2 - 2x - 4y - 5 = 0$ has centre C. A straight line with equation $y = 2x + 5$ intersects the circle at two points, A and B.

(i) Determine the coordinates of C.

[3 marks]

(ii) Show that the coordinates of A and B are (0, 5) and (–2, 1) and that BC is perpendicular to AC.

[8 marks]

(iii) Determine the equation of the tangent to the circle at either Point A or Point B.

[2 marks]

(b) The points $P(3, -1, 2)$, $Q(1, 2, -4)$ and $R(-1, 1, -2)$ are three vertices of a parallelogram $PQRS$.

(i) Express the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ in the form $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

[3 marks]

(ii) Show that the vector $\mathbf{s} = -16\mathbf{j} - 8\mathbf{k}$ is perpendicular to the plane through P, Q and R.

[5 marks]

(iii) Hence, determine the Cartesian equation of the plane through P, Q and R.

[4 marks]

Total 25 marks

SECTION C

Module 3

Answer BOTH questions.

5. (a) A function is given as $f(x) = \begin{cases} \sqrt{-x} & x < 0 \\ 1 & 0 < x \leq 1 \\ \sqrt{x} & x > 1. \end{cases}$

(i) Determine $\lim_{x \to 0^-} f(x)$.

[2 marks]

(ii) Determine whether $\lim_{x \to 0} f(x)$ exists. **Give a reason for your answer.**

[3 marks]

(iii) Determine whether f is continuous at $x = 1$. **Give a reason for your answer.**

[3 marks]

(b) Determine the point at which the tangent to the curve $y = x\sqrt{x}$ is parallel to the line $3x - y + 6 = 0$.

[4 marks]

(c) A function is given as $y = \sin^2(\cos x)$.

(i) Show that the x-coordinates of the stationary values of y are

- $n\pi, n \in \mathbf{Z}$
- $\pm \frac{\pi}{2} + 2n\pi$, n $\in \mathbf{Z}$.

[9 marks]

(ii) Determine the nature of the stationary values at $x = n\pi$, $n \in \mathbf{Z}$.

[4 marks]

Total 25 marks

6. (a) By using the substitution $u = 1 - x$, determine $\int x(1 - x)^2 \, dx$.

[5 marks]

(b) Given that $f(t) = 2 \cos t$ and $g(t) = 4 \sin 5t + 3 \cos t$, show that

$$\int [f(t) + g(t)]\, dt = \int f(t)\, dt + \int g(t)\, dt.$$

[5 marks]

(c) Calculate the volume of the solid formed by rotating the region enclosed by $y = \sin x$, $x = \frac{\pi}{2}$, $x = \pi$ and $y = 0$ about the x-axis.

[6 marks]

(d) Solve the differential equation $\frac{dy}{dx} = \frac{x\sqrt{x^2+1}}{y}$, given that when $x = 0, y = 2$.

[9 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.