CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

23 MAY 2019 (p.m.)

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FILL IN ALL THE INFORMATION REQUESTED CLEARLY IN CAPITAL LETTERS.

TEST CODE | 0 | 2 | 2 | 3 | 4 | 0 | 2 | 0 |

SUBJECT PURE MATHEMATICS – UNIT 2 – Paper 02

PROFICIENCY ADVANCED

REGISTRATION NUMBER

SCHOOL/CENTRE NUMBER

NAME OF SCHOOL/CENTRE

CANDIDATE'S FULL NAME (FIRST, MIDDLE, LAST)

DATE OF BIRTH | D | D | M | M | Y | Y | Y | Y |

SIGNATURE ____________________

TEST CODE **02234020**

FORM TP 2019308

MAY/JUNE 2019

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 2 – Paper 02

ANALYSIS, MATRICES AND COMPLEX NUMBERS

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of TWO questions.
3. Answer ALL questions from the THREE sections.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

<u>Examination Materials Permitted</u>

Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A

Module 1

Answer BOTH questions.

1. (a) Let $4x^2 + 3xy^2 + 7x + 3y = 0$.

(i) Use implicit differentiation to show that $\frac{dy}{dx} = -\frac{8x + 3y^2 + 7}{3(1 + 2xy)}$.

[5 marks]

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(ii) Show that for $f(x, y) = 4x^2 + 3xy^2 + 7x + 3y$,

$$6\frac{\partial f(x,y)}{\partial y} - 10 = \left(\frac{\partial^2 f(x,y)}{\partial y^2}\right)\left(\frac{\partial^2 f(x,y)}{\partial y\,\partial x}\right) + \frac{\partial^2 f(x,y)}{\partial x^2}.$$

[5 marks]

(b) Use de Moivre's theorem to prove that $\sin 5x = 16 \sin^5 x - 20 \sin^3 x + 5 \sin x$.

[6 marks]

(c) (i) Write the complex number $z = (-1 + \sqrt{3}\, i)^7$ in the form $re^{i\theta}$, where $r = |z|$ and $\theta = \arg z$.

[3 marks]

(ii) Hence, prove that $(-1 + \sqrt{3}\ i)^7 = 64\ (-1 + \sqrt{3}\ i)$.

[6 marks]

Total 25 marks

2. (a) Let $F_n(x) = \int (\ln x)^n \, dx$.

(i) Show that $F_n(x) = x (\ln x)^n - nF_{n-1}(x)$.

[3 marks]

(ii) Hence, or otherwise, show that $F_3(2) - F_3(1) = 2(\ln 2)^3 - 6(\ln 2)^2 + 12 \ln 2 - 6$.

[7 marks]

(b) (i) By expressing $\frac{y^2 + 2y + 1}{y^4 + 2y^2 + 1}$ as partial fractions, show that

$$\frac{y^2 + 2y + 1}{y^4 + 2y^2 + 1} = \frac{1}{y^2 + 1} + \frac{2y}{(y^2 + 1)^2}.$$

[7 marks]

(ii) Hence, or otherwise, evaluate $\int \frac{y^2 + 2y + 1}{y^4 + 2y^2 + 1}\, dy$.

[8 marks]

Total 25 marks

[8 marks]

Total 25 marks

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SECTION B

Module 2

Answer BOTH questions.

3. (a) Determine the coefficient of the term in x^3 in the binomial expansion of $(3x + 2)^5$.

[3 marks]

(b) (i) Show that the binomial expansion of $(1 + x)^{1/4} + (1 - x)^{1/4}$ up to the term in x^2 is

$2 - \frac{3}{16}x^2$.

[4 marks]

(ii) Hence, by letting $x = \frac{1}{16}$, compute an approximation of $\sqrt[4]{17} + \sqrt[4]{15}$, correct to 4 decimal places.

[3 marks]

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(c) The function $h(x) = x^3 + x - 1$ is defined on the interval [0, 1].

(i) Show that $h(x) = 0$ has a root on the interval [0, 1].

[3 marks]

(ii) Use the iteration $x_{n+1} = \dfrac{1}{x_n^2 + 1}$ with initial estimate $x_1 = 0.7$ to estimate the root of $h(x) = 0$, correct to 2 decimal places.

[6 marks]

(d) Use the Newton–Raphson method with initial estimate $x_1 = 5.5$ to approximate the root of $g(x) = \sin 3x$ in the interval [5, 6], correct to 2 decimal places.

[6 marks]

Total 25 marks

4. (a) A function is defined as $g(x) = x \sin\left(\frac{x}{2}\right)$.

(i) Obtain the Maclaurin series expansion for g up to the term in x^4.

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[8 marks]

(ii) Hence, estimate $g(2)$.

[2 marks]

(b) A series is given as $2 + \frac{3}{4} + \frac{4}{9} + \frac{5}{16} + \ldots$

(i) Express the n^{th} partial sum S_n of the series using sigma notation.

[2 marks]

(ii) Hence, calculate $S_{20} - S_{18}$.

[1 mark]

(iii) Given that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, show that S_n diverges.

[4 marks]

(c) Use the method of induction to prove that $\sum_{r=1}^{n} r(r-1) = \frac{n(n^2-1)}{3}$.

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[8 marks]

Total 25 marks

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[8 marks]

Total 25 marks

SECTION C

Module 3

Answer BOTH questions.

5. (a) (i) How many numbers made up of **five** digits can be made from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 if each number contains exactly one even digit and no digit is repeated?

[4 marks]

(ii) Determine the probability that the number formed in (a) (i) is less than 30 000.

[4 marks]

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(b) A and B are two matrices given as

$$A = \begin{pmatrix} 2 & x & -1 \\ 3 & 0 & 2 \\ 2 & 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 4 \\ 2 & 1 & 2 \end{pmatrix}.$$

(i) Determine the value of x for which A^{-1} does NOT exist.

[4 marks]

(ii) Given that $\det(AB) = -10$, show that $x = 2$.

[4 mark

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(iii) Hence, obtain A^{-1}.

[4 marks]

(c) In an experiment, individuals were asked to select from two available colours, green and blue. The individuals selected one colour, two colours or no colour.

70% of the individuals selected at least one colour and 600 individuals selected no colour.

(i) Given that 40% of the individuals selected green and 50% selected blue, calculate the probability that an individual selected BOTH colours.

[3 marks]

(ii) Determine the total number of individuals who participated in the experiment.

[2 marks]

Total 25 marks

6. (a) A differential equation is given as $x\dfrac{dy}{dx} + y = 2\sin x$.

(i) Show that the general solution of the differential equation is $y = \dfrac{c}{x} - \dfrac{2}{x}\cos x$, where c is a constant.

[5 marks]

(ii) Hence, determine the particular solution of the differential equation that satisfies the condition $y = 2$ when $x = \pi$.

[3 marks]

(b) Show that the general solution of the differential equation $\frac{dy}{dx} = \frac{xy - y}{x^2 - 4}$ is $y = k\sqrt[4]{(x-2)(x+2)^3}$, where k is a constant.

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[7 marks]

(c) Solve the boundary-value problem $y'' - y' - 2y = 0$, given that when $x = -1$, $y = 1$ and when $x = 1$, $y = 0$.

[10 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No. ☐