FORM TP 2021327

TEST CODE **02134020**

MAY/JUNE 2021

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 02

ALGEBRA, GEOMETRY AND CALCULUS

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of TWO questions.
3. Answer ALL questions from the THREE sections.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

<u>**Examination Materials Permitted**</u>

Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A

Module 1

Answer BOTH questions.

1. (a) (i) Let **p** and **q** be any two propositions. Complete the truth table below.

p	q	~p	~q	p ∨ q	~(p ∨ q)	~p ∨ ~q
T	T					
T	F					
F	T					
F	F					

[4 marks]

(ii) Hence, state whether the statements ~(**p** ∨ **q**) and ~**p** ∨ ~**q** are logically equivalent. **Justify your response.**

....

....

[2 marks]

(b) (i) Write the converse of the statement "**n is an integer** $\Rightarrow$ **n^2 is an integer**".

....

....

[1 mark]

(ii) For two real numbers x and y, the operation * is given by $x * y = x^2 + y^2$. Prove that * is closed in **R**.

[3 marks]

(iii) Determine whether the operation * is commutative.

[2 marks]

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(c) The function $f(x) = ax^3 + 3x^2 - b$ is divisible by $2x - 1$ and has a remainder of -5 when divided by $x + 2$.

Calculate the values of a and b.

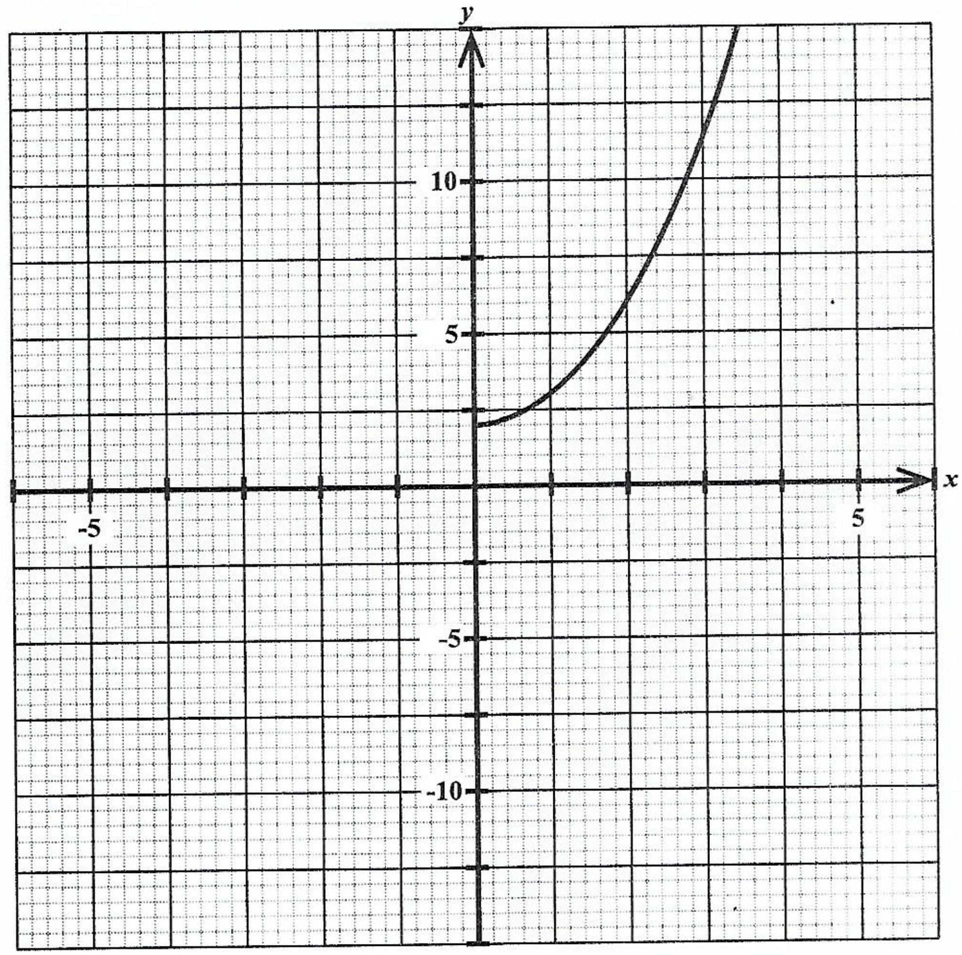
[7 marks]

(d) Solve the logarithmic equation $\log_2 x + \log_4 x + \log_{16} x = 7$.

[6 marks]

Total 25 marks

2. (a) The diagram below shows the graph of $f(x) = x^2 + 2$ for $x \geq 0$.



(i) On the graph

a) Sketch the inverse of f

b) Show that the inverse of f is a function. **[4 marks]**

(ii) Prove that f is one to one.

[4 marks]

(iii) Determine whether f is onto.

[3 marks]

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(b) Solve the equation $\left| x^2 - 4 \right| = 3x - 2.$

[6 marks]

(c) Given that α, β, and γ are the roots of the equation $2x^3 - x^2 + 3x - 1 = 0$, determine the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$.

[8 marks]

Total 25 marks

SECTION B

Module 2

Answer BOTH questions.

3. (a) (i) Express $4 \sin \theta + 3 \cos \theta$ in the form of $R \sin(\theta + \alpha)$.

[5 marks]

(ii) Hence, solve the equation $4 \sin \theta + 3 \cos \theta = 2$ for $0 \leq \theta \leq 2\pi$.

[5 marks]

(b) Given that $\cos A = \frac{12}{13}$, $\sin C = \frac{5}{13}$, where A and B are acute angles, calculate the value of $\cos (A - C)$.

[7 marks]

(c) Solve the equation $\cos\theta = \sin\left(\theta - \frac{\pi}{3}\right)$ for θ.

[8 marks]

Total 25 marks

4. (a) A circle has equation $x^2 + y^2 - 4x + 10y - 8 = 0$.

(i) Determine the centre and radius of the circle.

[4 marks]

(ii) Determine the equation of the tangent which touches the circle at (3, 11).

[4 marks]

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(b) Show that the curve whose parametric equations are $x = 3 + 3 \sin \theta$ and $y = 3 \cos \theta$ represents a circle.

[5 marks]

(c) Two position vectors are represented by $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$.

Determine the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

[8 marks]

(d) Determine the vector equation of a plane which passes through the point (2, 5, 3) and is perpendicular to the vector $4\mathbf{i} + 4\mathbf{j} - \mathbf{k}$.

[4 marks]

Total 25 marks

SECTION C

Module 3

Answer BOTH questions.

5. (a) Use the substitution $u = (x^3 + 4)$ to determine $\int 3x^2 (x^3 + 4)^4 \, dx$.

[6 marks]

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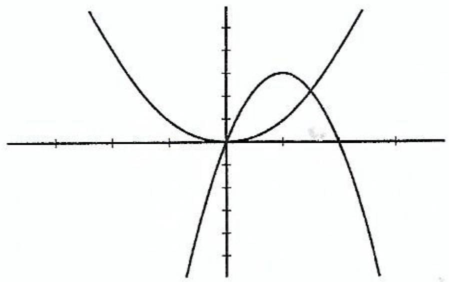
(b) Calculate the area of the region enclosed by the parabolas $y = x^2$ and $y = 6x - 3x^2$.

[A sketch of the curves is shown below.]



[9 marks]

(c) The equation of a curve is given as $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$.

(i) Show that $f(x)$ has stationary points at $x = -1$, $x = 0$ and $x = 2$.

[6 marks]

(ii) Determine the nature of these stationary points.

[4 marks]

Total 25 marks

6. (a) A function f is defined as $f(x) = \begin{cases} 2x + 3; x < 1 \\ 2 \qquad ; x = 1 \\ \dfrac{x^2 - 1}{x - 1}; x > 1 \end{cases}$

(i) Determine whether or not the $\lim\limits_{x \to 1} f(x)$ exists.

[5 marks]

(ii) Determine whether f is continuous at $x = 1$.

[2 marks]

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(b) The parametric equation of a curve is given by $x = 5 \cos \theta$ and $y = 2 \sin \theta$.

Determine $\frac{dy}{dx}$ in terms of θ.

[4 marks]

(c) Given that $\int_1^4 f(x)\,dx = \frac{10}{3}$, $\int_1^2 f(x)\,dx = 2$ and $g(x) = +\sqrt{x}$, calculate $\int_2^4 [f(x) + g(x)]\,dx$.

[4 marks]

(d) (i) Solve the differential equation $\frac{dy}{dx} = \frac{\sin x}{\sin y}$ given that when $x = 0$, $y = \frac{\pi}{2}$.

[6 marks]

(ii) Determine the equation of the curve that passes through (1, 5) and for which $y = \int 6x^2\,dx$.

[4 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.