

FORM TP 2022330



TEST CODE **02234020**

MAY/JUNE 2022

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 2 – Paper 02

ANALYSIS, MATRICES AND COMPLEX NUMBERS

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of TWO questions.
3. Answer ALL questions from the THREE sections.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – Revised 2012

Mathematical instruments

Silent, non-programmable electronic calculator

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SECTION A

Module 1

Answer BOTH questions.

1. (a) Using DeMoivre's theorem, prove that $\frac{\sin 5\theta}{\sin \theta} = 5 - 20 \sin^2 \theta + 16 \sin^4 \theta$.

[7 marks]

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- (b) Given that $3 + 5i$ is a root of the quadratic equation $z^2 + pz + q = 0$, determine the values of $p, q \in \mathbf{R}$.

[7 marks]

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- (c) A complex number, z , is such that $\arg(z - 2) = \frac{\pi}{2}$ and $\arg(z) = \frac{\pi}{3}$. Determine the complex number z .

[5 marks]

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- (d) Given that $(x^2 + y^2)^2 = ax^2y$, use implicit differentiation to determine $\frac{dy}{dx}$.

[6 marks]

Total 25 marks

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2. (a) (i) Use partial fractions to show that $\frac{x^4}{x^4 - 1} = 1 + \frac{1}{4(x - 1)} - \frac{1}{4(x + 1)} - \frac{1}{2(x^2 + 1)}$.

[9 marks]



- (ii) Hence, or otherwise, determine $\int \frac{x^4}{x^4 - 1} dx$.

[6 marks]



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- (b) (i) Show that if $I_n = \int (1-x)^n e^{ax} dx$ for $n \geq 1$, then $I_n = \frac{1}{a} (1-x)^n e^{ax} + \frac{n}{a} I_{n-1}$.

[5 marks]

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(ii) Hence, determine $\int (1-x)^3 e^{2x} dx$.

[5 marks]

Total 25 marks

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SECTION B

Module 2

Answer BOTH questions.

3. (a) (i) Use mathematical induction to prove that

$$\frac{3}{4} + \frac{5}{36} + \dots + \frac{2n-1}{n^2(n-1)^2} = 1 - \frac{1}{n^2}, n \geq 1.$$

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[9 marks]

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(ii) Hence, or otherwise, calculate $S_{30} - S_{10}$.

[4 marks]



- (b) Calculate the sum to infinity of the series $\sum_{r=2}^{\infty} \frac{10}{r^2 - 1}$.

[7 marks]

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- (c) Determine the Taylor series expansion of $x \sin\left(\frac{x}{2}\right)$, about $x = \pi$, up to and including the **first THREE** non-zero terms.

[5 marks]

Total 25 marks

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4. (a) (i) Show that the binomial expansion of $(1 + 5x)^{\frac{1}{5}}$ up to and including the term in x^3 is $1 + x - 2x^2 + 6x^3$.

[6 marks]

- (ii) Hence, by letting $x = -\frac{1}{32}$, compute an estimate of $\sqrt[5]{27}$.

[5 marks]

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- (b) (i) Use the Intermediate Value Theorem to prove that $4e^x + 2x^2 - 5 = 0$ has a root in the interval $[0, 1]$.

[3 marks]

- (ii) Use **four** iterations of the interval bisection method to calculate an approximation of the root of $4e^x + 2x^2 - 5 = 0$ in the interval $[0, 1]$.

[6 marks]

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- (iii) Use **three** iterations of the linear interpolation method to approximate the root of $4e^x + 2x^2 - 5 = 0$ in the interval $(-2, -1)$.

[5 marks]

Total 25 marks

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SECTION C

Module 3

Answer BOTH questions.

5. (a) A matrix \mathbf{P} is given as $\mathbf{P} = \begin{bmatrix} 4 & 3 & 5 \\ 9 & 4 & 15 \\ 12 & 10 & -3 \end{bmatrix}$.

(i) Show that $|\mathbf{P}| = 183$.

[4 marks]



- (ii) Hence, or otherwise, show that the adjoint of \mathbf{P} is $\text{adj}(\mathbf{P}) = \begin{bmatrix} -162 & 59 & 25 \\ 207 & -72 & -15 \\ 42 & -4 & -11 \end{bmatrix}$.

[5 marks]

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(iii) Solve the system of linear equations

$$\begin{bmatrix} 4 & 3 & 5 \\ 9 & 4 & 15 \\ 12 & 10 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 13 \\ 4 \end{bmatrix}.$$

[8 marks]

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- (b) Alex has five blue marbles, four green marbles and three red marbles. In how many ways can he arrange four marbles in a row, if the marbles of any given colour are identical?

[4 marks]

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- (c) Let A and B be two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{8}$. Calculate the value of $P(A' \cup B')$.

[4 marks]

Total 25 marks

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6. (a) The table below shows the probability distribution of colour-blind men and women in a population, where 51% of the population are male and 49% are female. The event that a person is colour-blind is A and the event that a person is a male is B .

	Men (B)	Women (B')	
Colour-blind (A)	0.04	0.002	0.042
Not Colour-blind (A')	0.47	0.488	0.958
Total	0.51	0.49	1.00

- (i) Determine $P(A | B)$.

[2 marks]

- (ii) Calculate the probability that a member of the population, selected at random, is colour-blind.

[2 marks]

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- (b) Solve the initial value problem $x^2 \frac{dy}{dx} + 2xy = \cos x$, where $y(\pi) = 0$.

[9 marks]

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- (c) Determine the general solution of the differential equation

$$y'' - 7y' + 12y = \sin x - \cos x.$$



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[12 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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