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FORM TP 2015037

MAY/JUNE 2015

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN SECONDARY EDUCATION CERTIFICATE®
EXAMINATION

ADDITIONAL MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

05 MAY 2015 (p.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of FOUR sections. Answer ALL questions in Section I, Section II and Section III.
2. Answer ONE question in Section IV.
3. Write your solutions with full working in the booklet provided.
4. A list of formulae is provided on page 2 of this booklet.

Required Examination Materials

Electronic Calculator (non-programmable)
Geometry Set
Mathematical Tables (provided)
Graph Paper (provided)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

LIST OF FORMULAE

Arithmetic Series	$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$
Geometric Series	$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_\infty = \frac{a}{1-r}, -1 < r < 1 \text{ or } \lvert r \rvert < 1$
Circle	$x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x+f)^2 + (y+g)^2 = r^2$
Vectors	$\hat{\mathbf{v}} = \frac{\mathbf{v}}{\lvert\mathbf{v}\rvert} \qquad \cos\theta = \frac{\mathbf{a}\bullet\mathbf{b}}{\lvert\mathbf{a}\rvert \times \lvert\mathbf{b}\rvert} \qquad \lvert\mathbf{v}\rvert = \sqrt{(x^2+y^2)}$ where $\mathbf{v} = x\mathbf{i} + y\mathbf{j}$
Trigonometry	$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$ $\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
Differentiation	$\frac{d}{dx}(ax+b)^n = an(ax+b)^{n-1}$ $\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\cos x = -\sin x$
Statistics	$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \qquad S^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\overline{x})^2$
Probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Kinematics	$v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2$

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SECTION I

Answer BOTH questions.

ALL working must be clearly shown.

1. (a) The functions f and g are defined by

$$f(x) = x^2 + 5, \qquad x \geq 1$$

$$g(x) = 4x - 3, \qquad x \in \mathbf{R}$$

where **R** is the set of real numbers.

Find the value of $g(f(2))$. **(2 marks)**

(b) The function h is defined by $h(x) = \dfrac{3x+5}{x-2}$ where $x \in \mathbf{R}$, $x \neq 2$.

Determine the inverse of $h(x)$. **(3 marks)**

(c) Given that $x - 2$ is a factor of $k(x) = 2x^3 - 5x^2 + x + 2$, factorize $k(x)$ completely. **(3 marks)**

(d) Solve the following equations:

(i) $16^{x+2} = \dfrac{1}{4}$ **(2 marks)**

(ii) $\log_3 (x + 2) + \log_3 (x - 1) = \log_3 (6x - 8)$ **(4 marks)**

Total 14 marks

2. (a) Given that $f(x) = 3x^2 - 9x + 4$:

(i) Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are real numbers. **(3 marks)**

(ii) State the coordinates of the minimum point of $f(x)$. **(1 mark)**

(b) The equation $3x^2 - 6x - 4 = 0$ has roots α and β. Find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$. **(4 marks)**

(c) Determine the coordinates of the points of intersection of the curve

$2x^2 - y + 19 = 0$ and the line $y + 11x = 4$. **(3 marks)**

(d) An employee of a company is offered an annual starting salary of \$36 000 which increases by \$2 400 per annum. Determine the annual salary that the employee should receive in the ninth year. **(3 marks)**

Total 14 marks

SECTION II

Answer BOTH questions.

ALL working must be clearly shown.

3. (a) The equation of a circle is given by $x^2 + y^2 - 12x - 22y + 152 = 0$.

(i) Determine the coordinates of the centre of the circle. **(2 marks)**

(ii) Find the length of the radius. **(1 mark)**

(iii) Determine the equation of the normal to the circle at the point (4, 10). **(3 marks)**

(b) The position vectors of two points, A and B, relative to an origin O, are such that

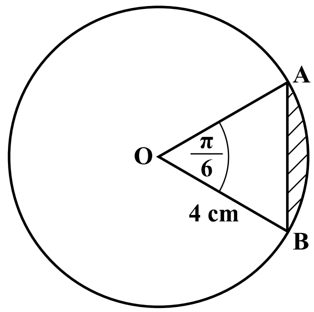
$\mathbf{OA} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{OB} = 5\mathbf{i} - 7\mathbf{j}$. Determine

(i) the unit vector **AB** **(3 marks)**

(ii) the acute angle AOB, in degrees, to one decimal place. **(3 marks)**

Total 12 marks

4. (a) The following diagram shows a circle of radius $r = 4$ cm, with centre O and sector AOB which subtends an angle, $\theta = \frac{\pi}{6}$ radians at the centre.



If the area of the triangle AOB $= \frac{1}{2} r^2 \sin \theta$, then calculate the area of the shaded region.

(4 marks)

(b) Solve the following equation, giving your answer correct to one decimal place.

$$8 \sin^2 \theta = 5 - 10 \cos \theta, \text{ where } 0^\circ \leq \theta \leq 360^\circ$$ **(4 marks)**

(c) Prove the identity

$$\frac{\sin \theta + \sin 2\theta}{1 + \cos \theta + \cos 2\theta} \equiv \tan \theta.$$ **(4 marks)**

Total 12 marks

SECTION III

Answer BOTH questions.

ALL working must be clearly shown.

5. (a) Differentiate the following expression with respect to x, simplifying your answer.

$(2x^2 + 3) \sin 5x$ **(4 marks)**

(b) (i) Find the coordinates of **all** the stationary points of the curve $y = x^3 - 5x^2 + 3x + 1$. **(3 marks)**

(ii) Determine the nature of EACH point in (i) above. **(2 marks)**

(c) A spherical balloon of volume $V = \frac{4}{3} \pi r^3$ is being filled with air at the rate of 200 $\text{cm}^3\ \text{s}^{-1}$.

Calculate**, in terms of π**, the rate at which the radius is increasing when the radius of the balloon is 10 cm. **(5 marks)**

Total 14 marks

6. (a) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 3 \cos \theta \, d\theta$. **(4 marks)**

(b) A curve has an equation which satisfies $\frac{dy}{dx} = kx(x - 1)$ where k is a constant.

Given that the value of the gradient of the curve at the point (2, 3) is 14, determine

(i) the value of k **(2 marks)**

(ii) the equation of the curve. **(4 marks)**

(c) Calculate, **in terms of $\boldsymbol{\pi}$**, the volume of the solid formed when the area enclosed by the curve $y = x^2 + 1$ and the x-axis, from $x = 0$ to $x = 1$, is rotated through 360° about the x-axis. **(4 marks)**

Total 14 marks

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SECTION IV

Answer only ONE question.

ALL working must be clearly shown.

7. (a) There are three traffic lights that a motorist must pass on the way to work. The probability that the motorist has to stop at the first traffic light is 0.2, and that for the second and third traffic lights are 0.5 and 0.8 respectively. Find the probability that the motorist has to stop at

(i) ONLY ONE one traffic light **(4 marks)**

(ii) AT LEAST TWO traffic lights. **(4 marks)**

(b) Use the data in the following table to estimate the mean of x.

x	5–9	10–14	15–19	20–24
f	8	4	10	3

(4 marks)

(c) Research in a town shows that if it rains on any one day then the probability that it will rain the following day is 25%. If it does **not** rain one day then the probability that it will rain the following day is 12%. Starting on a Monday and given that it rains on that Monday:

(i) Draw a probability tree diagram to illustrate the information, and show the probability on ALL of the branches. **(4 marks)**

(ii) Determine the probability that it will rain on the Wednesday of that week. **(4 marks)**

Total 20 marks

8. (a) A particle moving in a straight line has a velocity of 3 $m\ s^{-1}$ at $t = 0$ and 4 seconds later its velocity is 9 $m\ s^{-1}$.

(i) On the answer graph sheet, **provided as an insert**, draw a velocity–time graph to represent the motion of the particle. **(3 marks)**

(ii) Calculate the acceleration of the particle. **(3 marks)**

(iii) Determine the increase in displacement over the interval $t = 0$ to $t = 4$. **(4 marks)**

(b) A particle moves in a straight line so that t seconds after passing through a fixed point O, its acceleration, a, is given by $a = (3t - 1)m\ s^{-2}$. The particle has a velocity, v, of 4 $m\ s^{-1}$ when $t = 2$ and its displacement, s, from O is 6 metres when $t = 2$. Find

(i) the velocity when $t = 4$ **(5 marks)**

(ii) the displacement of the particle from O when $t = 3$. **(5 marks)**

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.