CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

04 JANUARY 2013 (a.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1.   This paper consists of TWO sections.

2.   There are EIGHT questions in Section I and THREE questions in Section II.

3.   Answer ALL questions in Section I, and any TWO questions from Section II.

4.   Write your answers in the booklet provided.

5.   All working must be clearly shown.

6.   A list of formulae is provided on page 2 of this booklet.

Required Examination Materials

Electronic calculator
Geometry set
Graph paper (provided)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

LIST OF FORMULAE

Volume of a prism	$V = Ah$ where A is the area of a cross-section and h is the perpendicular length.
Volume of cylinder	$V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.
Volume of a right pyramid	$V = \frac{1}{3} Ah$ where A is the area of the base and h is the perpendicular height.
Circumference	$C = 2\pi r$ where r is the radius of the circle.
Arc length	$S = \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended by the arc, measured in degrees.
Area of a circle	$A = \pi r^2$ where r is the radius of the circle.
Area of a sector	$A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle of the sector, measured in degrees.
Area of trapezium	$A = \frac{1}{2}(a + b)h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

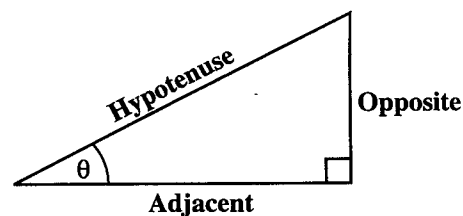Roots of quadratic equations   If $ax^2 + bx + c = 0$,

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Trigonometric ratios

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Area of triangle

Area of $\triangle = \frac{1}{2} bh$ where b is the length of the base and h is the perpendicular height.
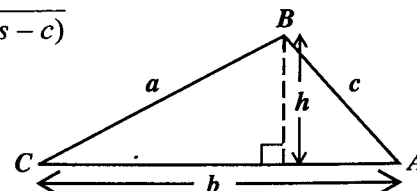
Area of $\triangle ABC = \frac{1}{2} ab \sin C$

Area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a + b + c}{2}$

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

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SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a)   Using a calculator or otherwise, calculate the exact value of

$$(2.67 \times 4.1) - 1.3^2.$$ **(3 marks)**

    (b)   Mr Harry who lives in St Kitts is planning to travel to Barbados. A travel club offers the rates shown below.

Petty's Travel Club	
Holiday in Barbados	
Return Air Fare	**US $356.00**
Hotel Accommodation	**US $97.00 per night**

    (i)   Calculate the TOTAL cost of airfare and hotel accommodation for 3 nights using the rates offered by Petty's Travel Club.   **(3 marks)**

    (ii)   Another travel club advertises the following package deal.

Angie's Travel Club
Holiday in Barbados
3 Nights Hotel Accommodation plus Return Air Fare
EC $1610.00

    Calculate, in US dollars, the cost of the trip for 3 nights as advertised by Angie's travel club.

    US $1.00 = EC $2.70   **(2 marks)**

    (iii) State, giving a reason for your answer, which travel club (Petty's or Angie's) has the better offer.   **(1 mark)**

    (iv)   The EC $1610.00 charged by Angie's Travel Club includes a sales tax of 15%. Calculate the cost of the trip for three nights BEFORE the sales tax was added.   **(2 marks)**

    Total 11 marks

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2. (a) Solve for p

$$2(p + 5) - 7 = 4p.$$ **(2 marks)**

(b) Factorize completely
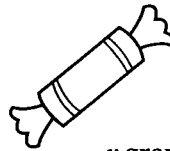
(i) $25m^2 - 1$ **(2 marks)**

(ii) $2n^2 - 3n - 20$ **(2 marks)**

(c) A candy store packages lollipops and toffees in bags for sale.

x grams y grams

5 lollipops and 12 toffees have a mass of 61 grams.
10 lollipops and 13 toffees have a mass of 89 grams.

(i) If the mass of one lollipop is x grams and the mass of one toffee is y grams, write two equations in x and y to represent the above information. **(2 marks)**

(ii) Calculate the mass of

a) ONE lollipop

b) ONE toffee **(4 marks)**

Total 12 marks

3. (a) There are 50 students in a class. Students in the class were given awards for Mathematics or Science.

36 students received awards in either Mathematics or Science.
6 students received awards in BOTH Mathematics and Science.
$2x$ students received awards for Mathematics only.
x students received awards for Science only.

In the Venn Diagram below:

U = {all the students in the class}
M = {students who received awards for Mathematics}
S = {students who received awards for Science}
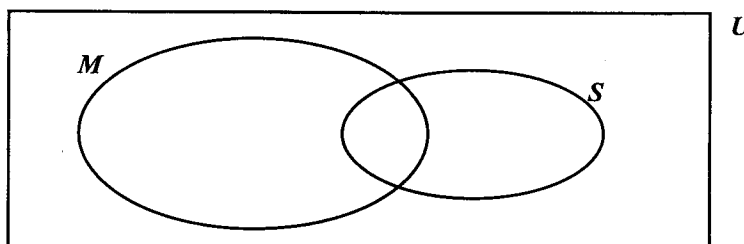
(i) Copy and complete the Venn Diagram to represent the information about the awards given, showing the number of students in EACH subset. **(4 marks)**

(ii) Calculate the value of x. **(2 marks)**

(b) In the diagram below, **not drawn to scale**, ABC is an isosceles triangle with $AB = AC$ and angle $ABC = 54°$. DE is parallel to BC.
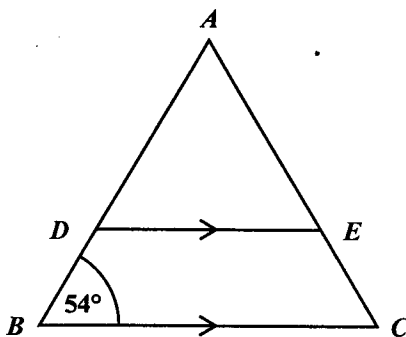
(i) Calculate, **giving a reason for your answer**, the measure of:

a) $\angle BAC$

b) $\angle AED$ **(4 marks)**

(ii) Explain why triangles ABC and ADE are similar but not congruent. **(2 marks)**

Total 12 marks

GO ON TO THE NEXT PAGE

4. (a) Make r the subject of EACH of the following formulae:

(i) $r - h = rh$ **(2 marks)**

(ii) $V = \pi r^2 h$ **(2 marks)**

(b) The functions f and g are defined as follows:

$$f(x) = 2x + 5$$

$$g(x) = \frac{x - 3}{2}$$

Evaluate:

(i) $f^{-1}(19)$ **(2 marks)**

(ii) $gf(3)$ **(2 marks)**

(c) A line segment GH has equation $3x + 2y = 15$.

(i) Determine the gradient of GH. **(1 mark)**

(ii) Another line segment, JK, is perpendicular to GH and passes through the point $(4, 1)$. Determine the equation of the line JK. **(3 marks)**
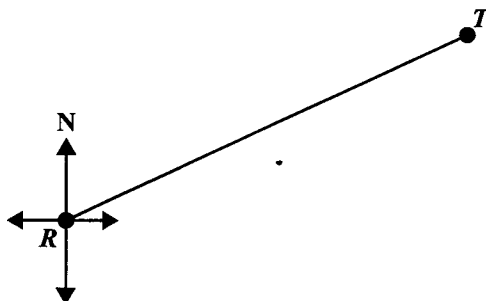
Total 12 marks

5. **An answer sheet is provided for this question.**

(a) The diagram below is a scale drawing showing the line *RT* and the north direction on a playground.

It is drawn to a scale of **1 centimetre : 30 metres**.

Using the answer sheet provided,

(i) measure and state, in centimetres, the length of *RT* as drawn on the diagram.

(1 mark)

(ii) measure and state, in degrees, the size of the angle that shows the bearing of *T* from *R*.

(2 marks)

(iii) calculate the actual distance, in metres, on the playground that *RT* represents.

(2 marks)

(b) A point *M* on the playground is located 300 metres from *R* on a bearing of 120°.

On the same answer sheet,

(i) calculate, in centimetres, the length of *RM* that should be used on the scale drawing.

(2 marks)

(ii) using a ruler and a pair of compasses, draw the line *RM* on the scale drawing.

(4 marks)

(iii) mark and name the angle in the scale drawing that measures 120°.   **(1 mark)**
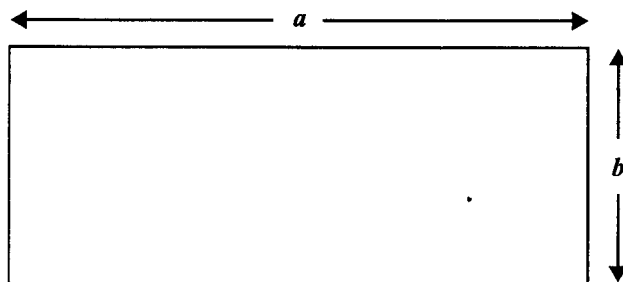
Total 12 marks

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6. The diagram below, **not drawn to scale**, shows a hollow cylinder with height 8 cm and diameter 12 cm.

Use π = 3.14

8 cm

12 cm
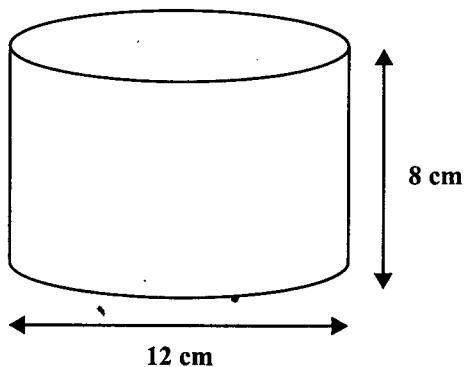
(a) Calculate for the cylinder:

   (i)   The radius   (1 mark)

   (ii) The circumference of the cross section   (2 marks)

(b) The rectangle shown below, **not drawn to scale**, represents the net of the curved surface of the cylinder shown above.

a

b

   (i)   State the values of a and b.   (2 marks)

   (ii) Hence, calculate the area of the curved surface of the cylinder.   (2 marks)

(c) If 0.5 litres of water is poured into the cylinder, calculate, correct to one decimal place, the height of water in the cylinder.   (4 marks)

Total 11 marks

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7. The scores obtained by 100 children in a competition are summarized in the table below.

Score	Class mid-point (x)	Frequency (f)	f × x
0–9	4.5	8	36
10–19	14.5	13	188.5
20–29		25	
30–39		22	
40–49		20	
50–59		12	
Total		100	

(a) (i) State the modal class interval. **(1 mark)**

(ii) State the class interval in which a score of 19.4 would lie. **(1 mark)**

(b) (i) Copy and complete the table to show

a) the class mid-points

b) the values of "f × x" **(2 marks)**

(ii) Calculate the mean score for the sample. **(3 marks)**

(c) Explain why the value of the mean obtained in (b) (ii) is only an estimate of the true value.
(1 mark)

(d) In order to qualify for the next round of the competition a student must score AT LEAST 40 points.

What is the probability that a student selected at random qualifies for the next round?
(2 marks)

Total 10 marks

8. The first three diagrams in a sequence are shown below.

$n = 1$ $n = 2$ $n = 3$
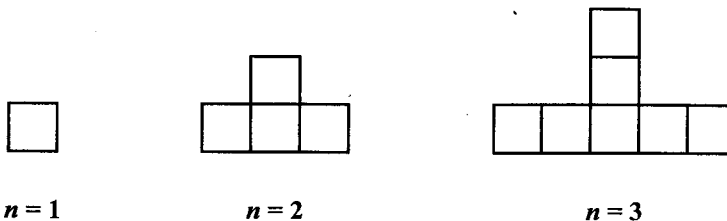
(a) In your answer booklet, draw the FOURTH diagram in the sequence. **(2 marks)**

(b) The table below shows the number of squares in EACH diagram.

	Diagram (n)	Number of Squares
	1	1
	2	4
	3	7
(i)	4	a
	- - -	- - -
(ii)	10	b
	- - -	- - -
(iii)	c	40

Determine the values of

(i) a

(ii) b

(iii) c **(5 marks)**

(c) Write down, in terms of n, the number of squares in the nth diagram of the sequence.

(3 marks)

Total 10 marks

SECTION II

Answer TWO questions.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9.   (a)   The table below shows corresponding values of x and y for the function

$$y = \frac{3}{x}, \qquad x \neq 0$$

where y represents the velocity of a particle after x seconds.

x (sec)	0.25	0.5	1	2	3	4	5	6
y (m/s)	12		3	1.5		0.75		0.5

        (i)   Copy and complete the table for the function.   **(2 marks)**

        (ii)   Using a scale of **2 cm to represent 1 unit on the x axis and 1 cm to represent 1 unit on the y axis**, plot the points from your table, drawing a smooth curve through all points.   **(5 marks)**

    (b)   (i)   Write $f(x) = 3x^2 - 5x + 1$ in the form $a(x - h)^2 + k$ where a, h and k are constants to be determined.   **(2 marks)**

        (ii)   Hence, or otherwise, determine the minimum value of $f(x)$ and the value of x for which $f(x)$ is a minimum.   **(2 marks)**

        (iii)   Solve the equation

        $3x^2 - 5x + 1 = 0$ expressing your answer correct to two decimal places.

        (4 marks)

Total 15 marks

GEOMETRY AND TRIGONOMETRY

10. (a) The diagram below, **not drawn to scale**, shows a circle, centre O. RQ is a diameter and PM and PN are tangents to the circle. Angle $MPN = 54°$ and angle $RQM = 20°$.
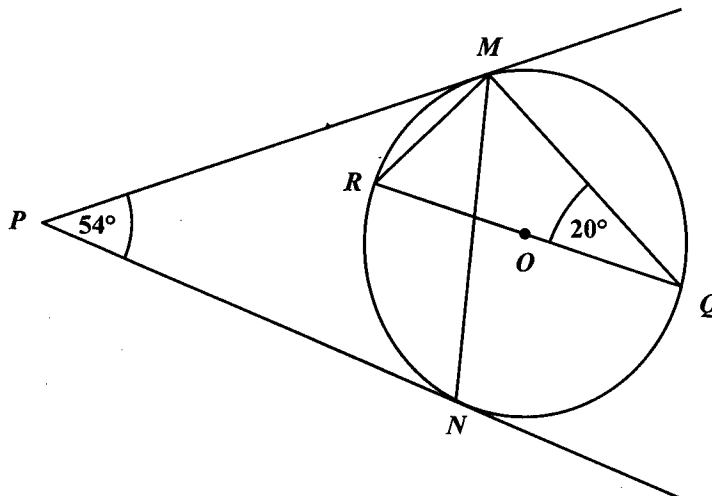
Calculate, giving reasons for your answer, the measure of:

(i) $\angle MRQ$ **(2 marks)**

(ii) $\angle PMR$ **(2 marks)**

(iii) $\angle PMN$ **(3 marks)**

(b) (i) The diagram below, **not drawn to scale**, shows the position of three points A, B and C on a horizontal plane.

$AB = 174$ metres, $BC = 65$ metres and $AC = 226$ metres
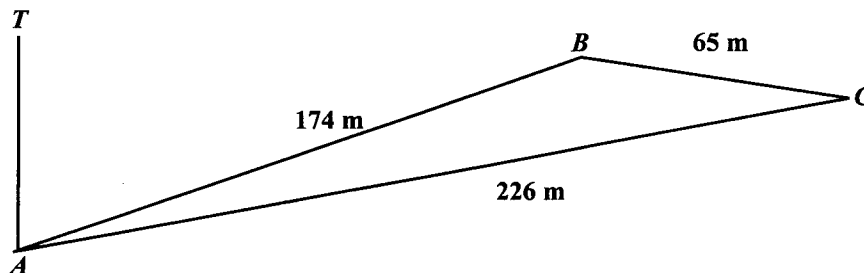
Calculate

a) the measure of angle ABC **(2 marks)**

b) the area of triangle ABC.   **(2 marks)**

(ii) The line *TA* represents a vertical lighthouse. The angle of elevation of *T* from *B* is 23°.

    a)   In your answer booklet, draw the triangle *TAB* showing the angle of elevation.

                                                      (2 marks)

    b)   Calculate the height, *TA*, of the lighthouse.   **(2 marks)**

                                                                Total 15 marks

VECTORS AND MATRICES

11.   **(a)**   The diagram below, **not drawn to scale**, shows a parallelogram *OKLM* where *O* is the origin. The point *S* is on *KM* such that $MS = 2\ SK$. $\overrightarrow{OK} = v$ and $\overrightarrow{OM} = u$.
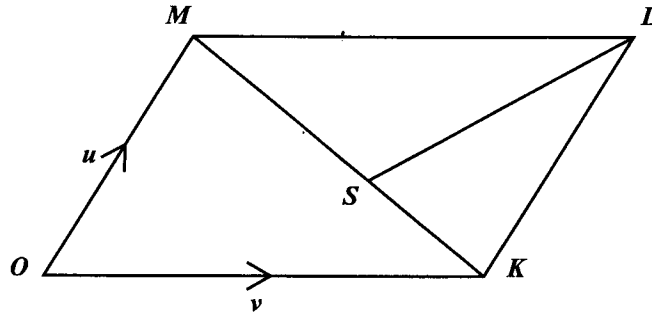
Express EACH of the following in terms of *u* and *v*:

(i) $\overrightarrow{MK}$ **(1 mark)**

(ii) $\overrightarrow{SL}$ **(2 marks)**

(iii) $\overrightarrow{OS}$ **(2 marks)**

(b)   The matrix $J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ represents a single transformation.

The image of the point *P* under transformation *J* is (5, 4).

Determine the coordinates of *P*.   **(3 marks)**

(c)   (i)   Write down a matrix, *H*, of size 2 x 2 which represents an enlargement of scale factor 3 about the origin.   **(1 mark)**

(ii)   Determine the coordinates of the point (5, −7) under the combined transformation, *H* followed by *J*.   **(2 marks)**

(d)   A superstore sells 3 models of cell phones. Model A costs $40 each, model B costs $55 each and model C costs $120 each.

The weekly sales for 2 weeks in June were:

Week 1	Week 2
2 model A	no model A
5 model B	6 model B
3 model C	10 model C

(i)   Write down a matrix of size 3 x 2 which represents the sales for the two weeks.

(1 mark)

(ii)   Write down a matrix of size 1 x 3 which represents the cost of the different models of cell phones.   **(1 mark)**

(iii) Write down the multiplication of the two matrices which represents the superstore's takings from the sale of cell phones for each of the two weeks.   **(2 marks)**

Total 15 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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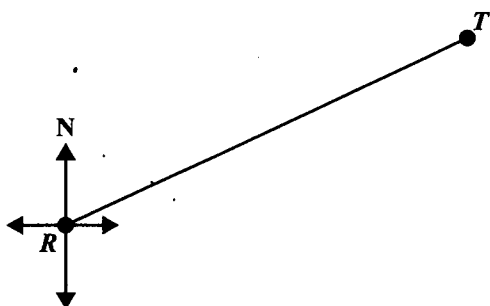CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MATHEMATICS

Paper 02 – General Proficiency

Answer Sheet for Question 5.

Candidate Number ....

(a)

Scale: 1 cm represents 30 m

    (i) RT = _____ centimetres

    (ii)   Angle = _____ degrees

    (iii)   Actual distance of RT = _____ metres

(b)   (i) RM = _____ centimetres

ATTACH THIS ANSWER SHEET TO YOUR ANSWER BOOKLET