TEST CODE **01234020**

FORM TP 2014089

MAY/JUNE 2014

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

21 MAY 2014 (a.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of TWO sections: I and II.
2. Section I has EIGHT questions and Section II has THREE questions.
3. Answer ALL questions in Section I, and any TWO questions from Section II.
4. Write your answers in the booklet provided.
5. Begin EACH question on a separate page.
6. All working must be shown clearly.
7. A list of formulae is provided on page 2 of this booklet.

Required Examination Materials

Electronic calculator
Geometry set
Graph paper (provided)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, determine the value of:

(i) $5.25 \div 0.015$ **(1 mark)**

(ii) $\sqrt{6.5025}$ **(1 mark)**

(iii) 3.142×2.236^2 (correct to 3 significant figures) **(2 marks)**

(b) Concrete tiles are made using buckets of cement, sand and gravel mixed in the ratio 1:4:6.

(i) How many buckets of gravel are needed for 4 buckets of cement? **(2 marks)**

(ii) If 20 buckets of sand are used, how many buckets of EACH of the following will be needed?

a) Cement

b) Gravel **(3 marks)**

(c) The cash price of a laptop is \$1 299. It can be bought on hire purchase by making a deposit of \$350 and 10 monthly payments of \$120 each.

(i) What is the TOTAL hire purchase price of the laptop? **(2 marks)**

(ii) How much is saved by buying the laptop for cash? **(1 mark)**

Total 12 marks

2. (a) Write as a single fraction in its LOWEST terms

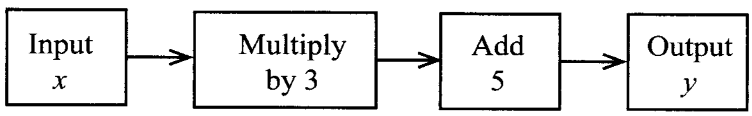
$$\frac{x-2}{3} + \frac{x+1}{4}.$$ **(3 marks)**

(b) Write an equation in x to represent EACH statement below. Do NOT solve the equation.

(i) When 4 is added to a certain number the result is the same as halving the number and adding 10. **(1 mark)**

(ii) Squaring a number and subtracting 6 gives the same result as doubling the number and adding 9. **(1 mark)**

(c) John drew the diagram below to show what he was thinking.



(i) Use information from the diagram to write a formula for y in terms of x. **(1 mark)**

(ii) If the number 4 is the input, what number would be the output? **(1 mark)**

(iii) If the number 8 was the output, what number was the input? **(1 mark)**

(iv) Reverse the formula written at (c) (i) above to write x in terms of y. **(1 mark)**

(d) Solve the following simultaneous equations:

$$2x + 3y = 9$$
$$3x - y = 8$$ **(3 marks)**

Total 12 marks

3. (a) The universal set, U, is defined as the set of integers between 11 and 26.

A and B are subsets of U such that:

$A = \{\text{even numbers}\}$

$B = \{\text{multiples of } 3\}$

(i) How many members are in the universal set, U? **(1 mark)**

(ii) List the members of the subset A. **(1 mark)**

(iii) List the members of the subset B. **(1 mark)**

(iv) Draw a Venn diagram to represent the relationships among A, B and U. **(3 marks)**

(b) (i) Using **a ruler, a pencil and a pair of compasses**, construct

a) a triangle PQR in which $PQ = 8$ cm, $PR = 6$ cm and angle $P = 60°$ **(3 marks)**

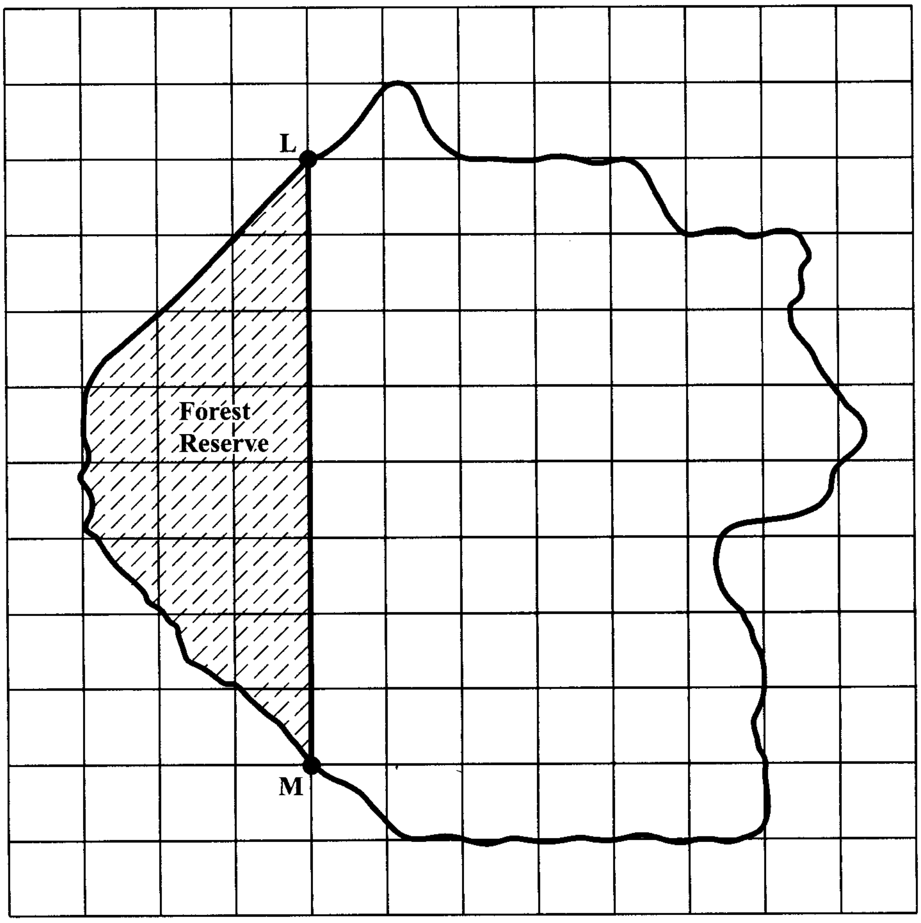
b) the line segment RX which is perpendicular to PQ and meets PQ at X. **(2 marks)**

[Note: Credit will be given for clearly drawn construction lines.]

(ii) Measure and state the size of angle QRX. **(1 mark)**

Total 12 marks

4. The diagram below shows a map of an island drawn on a grid of 1-cm squares. The map is drawn to a scale of 1:50 000.



(a) Copy and complete EACH of the following sentences:

(i) 1 cm on the map represents ____________ cm on the island. **(1 mark)**

(ii) An area of 1 cm^2 on the map represents an area of ____________ cm^2 on the island. **(1 mark)**

(iii) Given that 1 km = 100 000 cm, a distance of 1 cm on the map represents a distance of ____________ km on the island. **(1 mark)**

(b) (i) L and M are two tracking stations. State, in centimetres, the distance LM on the map. **(1 mark)**

(ii) Calculate the ACTUAL distance, in kilometres, from L to M on the island. **(2 marks)**

(c) (i) The area shaded on the map is a forest reserve. By counting squares estimate, in cm^2, the area of the forest reserve as shown on the map. **(2 marks)**

(ii) Calculate, in km^2, the ACTUAL area of the forest reserve. **(2 marks)**

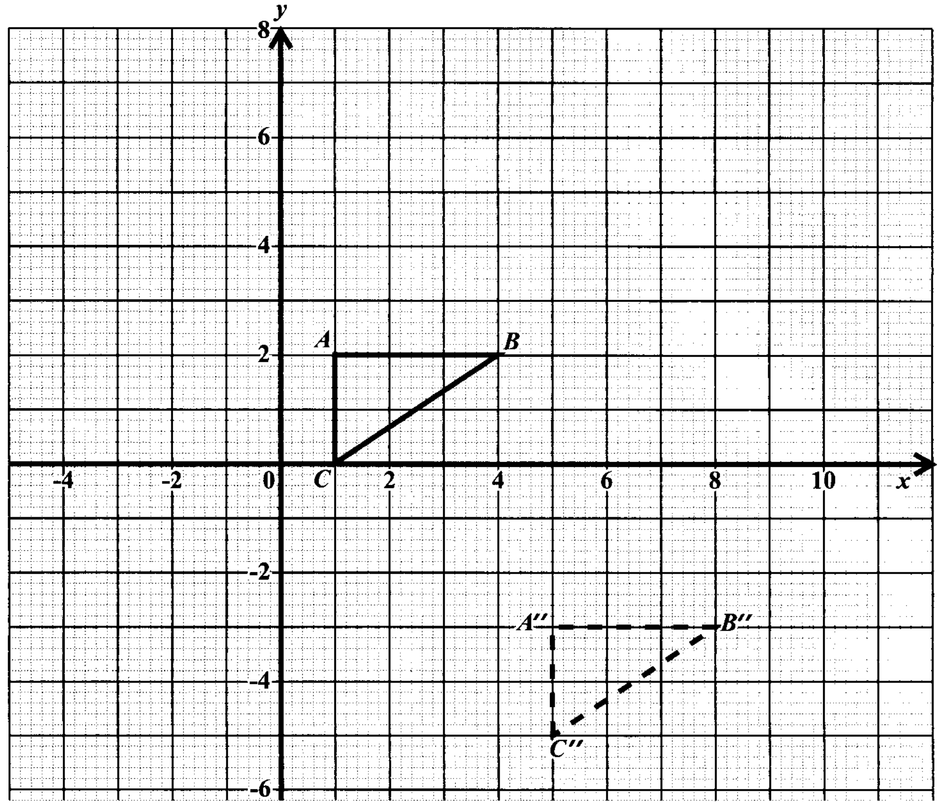
Total 10 marks

5. **An answer sheet is provided for this question.**

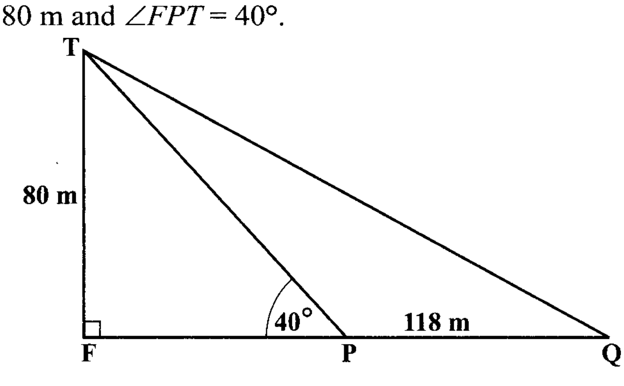
(a) Triangle ABC has coordinates $A(1,2)$, $B(4,2)$ and $C(1,0)$.

(i) **On the answer sheet provided**, draw triangle $A'B'C'$, the image of triangle ABC, under an enlargement, centre O and scale factor 2. **(3 marks)**

(ii) Triangle $A''B''C''$ is the image of triangle ABC, under a transformation, M. Describe **completely** the transformation, M. **(3 marks)**



(b) The diagram below, **not drawn to scale**, shows the positions of two ships, P and Q, at anchor. FT is the vertical face of a cliff jutting out of the water. P and Q are 118 m apart. $FT = 80$ m and $\angle FPT = 40°$.



Determine

(i) the angle of elevation of T from P **(1 mark)**

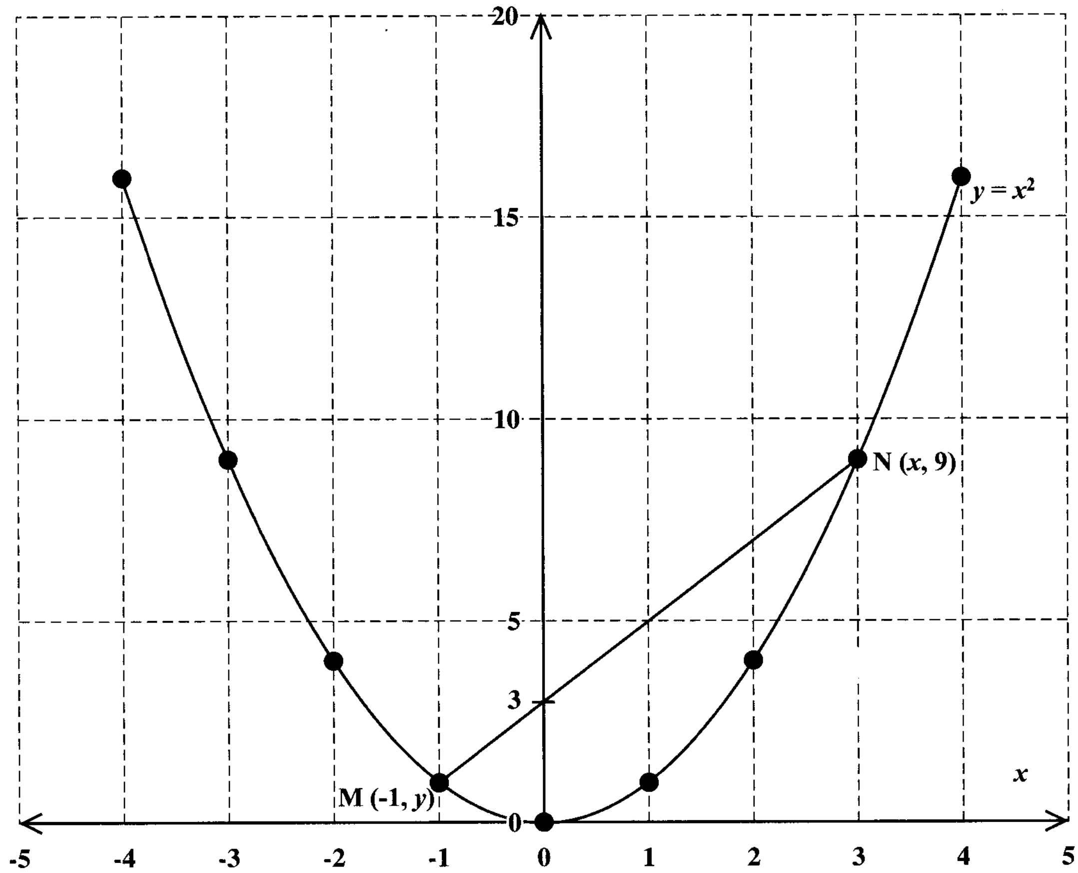
(ii) the length of FP **(2 marks)**

(iii) the angle of elevation of T from Q. **(3 marks)**

Total 12 marks

6. **An answer sheet is provided for this question.**

The graph of the quadratic function $y = x^2$ for $-4 \leq x \leq 4$ is shown below.



(a) The coordinates of the points M and N are $(-1, y)$ and $(x, 9)$ respectively.

Determine the value of

(i) x **(1 mark)**

(ii) y. **(1 mark)**

(b) Determine

(i) the gradient of the line MN **(1 mark)**

(ii) the equation of the line MN **(2 marks)**

(iii) the equation of the line parallel to MN, and passing through the origin. **(2 marks)**

(c) **On the answer sheet provided**, carefully draw the tangent line to the graph $y = x^2$ at the point (2, 4). **(2 marks)**

(d) Estimate the gradient of the tangent to the curve at (2, 4). **(2 marks)**

Total 11 marks

7. A class of 30 students counted the number of books in their bags on a certain day. The number of books in EACH bag is shown below.

5	4	6	3	2	1	7	4	5	3
6	5	4	3	7	6	2	5	4	5
5	7	5	4	3	2	1	6	3	4

(a) Copy and complete the frequency table for the data shown above.

Number of Books (x)	Tally	Frequency (f)	$f \times x$
1	||	2	2
2	|||	3	6
3			
4			
5			
6			
7			

(4 marks)

(b) State the modal number of books in the bags of the sample of students. **(1 mark)**

(c) Using the table in (a) above, or otherwise, calculate

(i) the TOTAL number of books **(2 marks)**

(ii) the mean number of books per bag. **(2 marks)**

(d) Determine the probability that a student chosen at random has LESS THAN 4 books in his/her bag. **(2 marks)**

Total 11 marks

8. **An answer sheet is provided for this question.**

A number sequence may be formed by counting the number of dots used to draw each of a set of geometric figures. The first three figures are shown below.

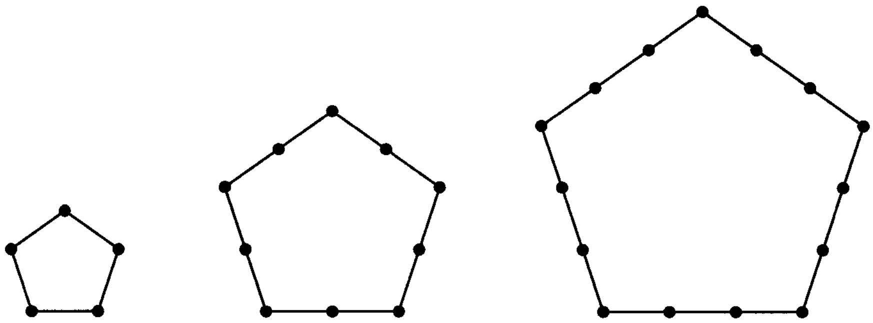


Figure 1 **Figure 2** **Figure 3**

On the answer sheet provided,

(a) Draw Figure 4, the next figure in the sequence above. **(2 marks)**

(b) Complete the table below by inserting the missing information at the rows numbered (i) and (ii).

	Figure (f)	Total Number of Dots	
		Formula	**Number (n)**
	1	$5 \times 2 - 5$	5
	2	$5 \times 3 - 5$	10
	3	$5 \times 4 - 5$	15
	4	Not Required	Not Required
(i)	5		
(ii)	6		

(4 marks)

(c) Write an expression in f for the number (n) of dots used in drawing the fth figure. **(2 marks)**

(d) Which figure in the sequence contains 145 dots? **(2 marks)**

Total 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) Two functions are defined as follows:

$$g(x) = 4x + 3$$

$$f(x) = \frac{2x + 7}{x + 1}$$

(i) State the value of x for which $f(x)$ is undefined. **(1 mark)**

(ii) Calculate the value of $gf(5)$. **(3 marks)**

(iii) Find $f^{-1}(x)$. **(3 marks)**

(b) A ball is thrown vertically upwards. Its height, h metres, above the ground after t seconds is shown in the table below.

t (s)	0	1	2	3	4	5	6
h (m)	0	50	80	90	80	50	0

(i) Using **2 cm to represent 1 second** on the x-axis and **1 cm to represent 10 metres** on the y-axis, plot a graph to show the height of the ball during the first 6 seconds. **(4 marks)**

(ii) **Using your graph**, determine

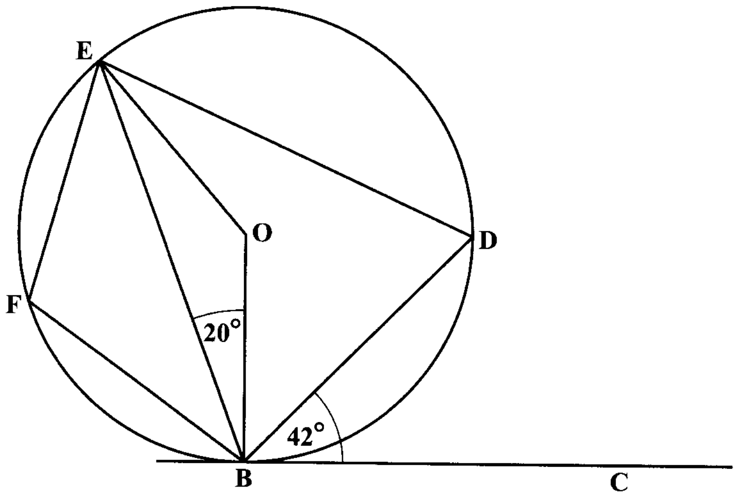
a) the average speed of the ball during the first 2 seconds **(2 marks)**

b) the speed of the ball when $t = 3$ seconds. **(2 marks)**

Total 15 marks

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) The diagram below, **not drawn to scale**, shows a circle, centre O. The line BC is a tangent to the circle at B. Angle $CBD = 42°$ and angle $OBE = 20°$.



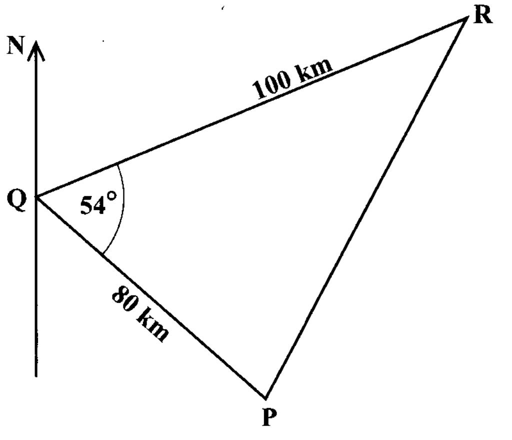
Calculate, giving a reason for EACH step of your answer, the measure of:

(i) $<BOE$ **(2 marks)**

(ii) $<OED$ **(2 marks)**

(iii) $<BFE$ **(3 marks)**

(b) The diagram below, **not drawn to scale**, shows the positions of three ports, P, Q and R.



Q is 80 km from P.

R is 100 km from Q on a bearing of 066°.

$<PQR = 54°$.

Calculate

(i) the bearing of P from Q **(2 marks)**

(ii) the distance PR correct to 2 decimal places **(3 marks)**

(iii) the measure of $<QPR$ to the **nearest** degree. **(3 marks)**

Total 15 marks

VECTORS AND MATRICES

11. (a) The matrix M is defined as

$$M = \begin{pmatrix} 7 & 2 \\ p & -1 \end{pmatrix}$$

Determine the value of p for which the matrix M does NOT have an inverse.

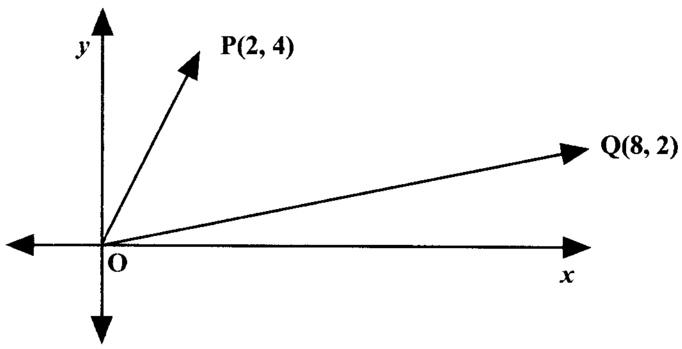
(2 marks)

(b) Express the equations

$4x - 2y = 0$
$2x + 3y = 4$

in the form $AX = B$, where A, X and B are matrices. **(2 marks)**

(c) In the diagram below, the coordinates of P and Q are (2, 4) and (8, 2) respectively. The line segment joining the origin (0, 0) to the point P may be written as $\overrightarrow{OP}$.



(i) What term is used to describe $\overrightarrow{OP}$? **(2 marks)**

(ii) Write EACH of the following in the form : $\begin{pmatrix} a \\ b \end{pmatrix}$

a) $\overrightarrow{OP}$ **(1 mark)**

b) $\overrightarrow{OQ}$ **(1 mark)**

c) $\overrightarrow{PQ}$ **(2 marks)**

(iii) Given that $\overrightarrow{OP} = \overrightarrow{RQ}$, determine the coordinates of the point, R. **(3 marks)**

(iv) State the type of quadrilateral formed by $PQRO$. **Justify your answer.**

(2 marks)

Total 15 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

LIST OF FORMULAE

Volume of a prism — $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of cylinder — $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

Volume of a right pyramid — $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference — $C = 2\pi r$ where r is the radius of the circle.

Arc length — $S = \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended by the arc, measured in degrees.

Area of a circle — $A = \pi r^2$ where r is the radius of the circle.

Area of a sector — $A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle of the sector, measured in degrees.

Area of trapezium — $A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations — If $ax^2 + bx + c = 0$,

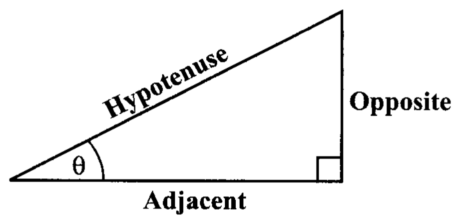
then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



Area of triangle — Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height.

Area of $\Delta ABC = \frac{1}{2}ab\sin C$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a+b+c}{2}$

Sine rule — $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule — $a^2 = b^2 + c^2 - 2bc\cos A$